Lecture 4

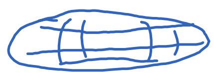
Wednesday, May 4, 2022 12:43 PM

* Prayer
* Spiritual thought

Quadratic surfaces

These are surfaces given by the equation $Ax^2+By^2+Cz^2+\dots+Iz+J=0$.

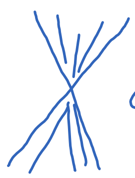
They boil down to the following basic shapes:



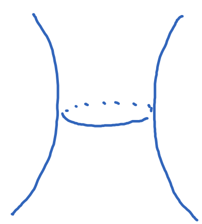
ellipsoid

elliptic paraboloid

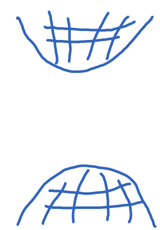
hyperbolic paraboloid



cone

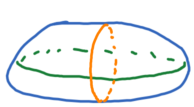


elliptic hyperboloid (one sheet)

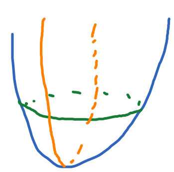


elliptic hyperboloid (two sheets)

The names come from the cross sections.



both cross sections are ellipse

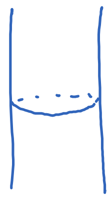


one cross section is ellipse, one is a parabola

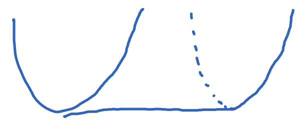
one cross section is parabola, one is a hyperbola

Name is ordered alphabetically: elliptic, hyperbolic, parabolic.

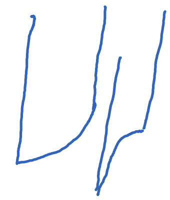
Cylinder



elliptic cylinder

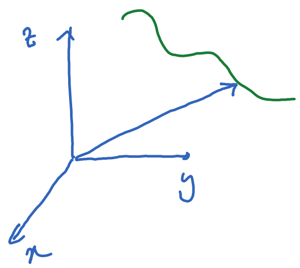


parabolic cylinder



hyperbolic cylinder

* Curves



A curve can be viewed as a multi-valued function $r(t)$ of one variable

$$r : (a, b) \to \mathbb{R}^3 \text{ or } \mathbb{R}^2$$

Ex

$$r(t) = (\cos t, \sin t)$$

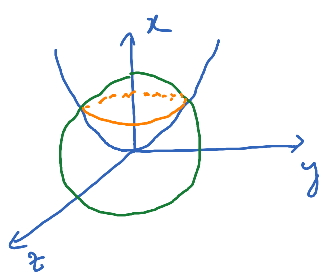
$$r(t) = (\cos t, \sin t, t)$$

$$r(t) = \left(\sqrt{1-t^2}\cos 5t, \sqrt{1-t^2}\sin 5t, t\right)$$

Use ParametricPlot command to plot.

$r(t)$ is called a parametrization of the curve.

Ex Parametrize the curve at the intersection between the elliptic paraboloid $x = y^2+z^2$ and the sphere $x^2+y^2+z^2 = 2$.

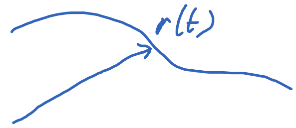


$$\begin{cases} x^2+y^2+z^2 = 2 \\ x = y^2+z^2 \end{cases} \longrightarrow \begin{cases} x^2+x = 2 \\ x = y^2+z^2 \end{cases} \longrightarrow \begin{cases} x = 1 \\ y^2+z^2 = 1 \end{cases}$$

$$r(t) = (1, \cos t, \sin t), \quad 0 \le t \le 2\pi.$$

Calculus of vector functions

$$r(t) = (x(t), y(t), z(t))$$



How to interpret $\lim_{t \to t_0} r(t)$?

$\lim_{t \to t_0} r(t) = (a, b, c)$ iff $\lim_{t \to t_0} x(t) = a$, $\lim_{t \to t_0} y(t) = b$, $\lim_{t \to t_0} z(t) = c$.

Ex

$$r(t) = \left(t \sin \frac{1}{t}, \frac{e^t - 1}{t}, 2t \right)$$

$$\lim_{t \to 0} r(t) = ?$$

$$r(t) = \left(\frac{1}{t-1}, t, 2 \right)$$

$$\lim_{t \to 0} r(t) = ?$$

Continuity: $r(t)$ is continuous at t_0 if $\lim_{t \to t_0} r(t) = r(t_0)$.

Derivative: How to interpret $r'(t)$?

One can think of $r'(t) = (x'(t), y'(t), z'(t))$ or

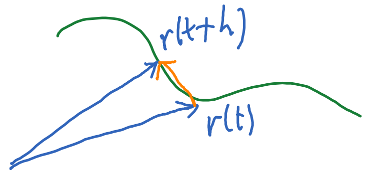
$$r'(t) = \lim_{h \to 0} \frac{r(t+h) - r(t)}{h}$$

The two ways are equivalent to each other.

$$r'(t) = \lim_{h \to 0} \frac{r(t+h) - r(t)}{h} = \lim_{h \to 0} \left(\frac{x(t+h) - x(t)}{h}, \frac{y(t+h) - y(t)}{h}, \frac{z(t+h) - z(t)}{h} \right)$$

$$= \left(\lim_{h \to 0} \frac{x(t+h) - x(t)}{h}, \dots, \dots \right) = \left(x'(t), y'(t), z'(t) \right).$$

<u>Geometric meaning of derivative:</u>



$r(t+h) - r(t) =$ orange vector, which goes to zero as $h \to 0$.

The direction of this vector tends to the direction of the tangent vector of the curve.

$\frac{r(t+h) - r(t)}{h} \rightsquigarrow$ stretching the orange vector by factor $\frac{1}{h}$.

$r'(t) = \lim_{h \to 0} \frac{r(t+h) - r(t)}{h}$ is a tangent vector to the curve.

<u>Ex</u> Find a tangent vector to the spiral curve $r(t) = (\cos t, \sin t, t)$ at $(1, 0, 0)$.

$r(0) = (1, 0, 0) \rightsquigarrow r'(0)$ is a tangent vector at $(1, 0, 0)$

$r'(t) = (-\sin t, \cos t, 1) \rightsquigarrow r'(0) = (0, 1, 1)$.